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Directionality of light transmission and reflection in two-dimensional Penrose tiled photonic quasicrystals

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Abstract
We have investigated the directionality in the reflection and transmission spectra of Penrose-type photonic quasicrystals. In the frequency interval considered, there are two omnidirectional stopbands. It is found that for the low-frequency stopband light is reflected ballistically, while for the higher-frequency band reflected light is scattered with a change in wavevector equal to one of the reciprocal lattice vectors associated with the first stopband. The relationship between the intensities of the ballistic and the scattered reflected and transmitted light suggests that two-dimensional quasicrystals possess an omnidirectional photonic bandgap for the E and H polarizations.

1. Introduction

Photonic crystals have attracted considerable interest in the last decade because of their ability to inhibit the spontaneous emission of light and their other potential applications in various optoelectronic devices [1–3].

For crystals, the highest level of rotational symmetry is six, and even in such structures the frequency range of bandgaps for different directions in the Brillouin zone can differ substantially. In quasicrystals [4, 5], which exhibit long-range but non-periodic order, the degree of rotational symmetry can be greater than for crystals, e.g. eight, ten, or even more. Initial theoretical and experimental studies had suggested the presence of omnidirectional photonic bandgaps for photonic quasicrystals [6–8], but since then some contradictions have emerged between the results obtained by different groups [9], and more detailed investigation of the properties of photonic quasicrystals is required.

Usually, the presence of a frequency within which transmission decays with an increase of the sample thickness is considered as proof of the bandgap for that particular direction. On the other hand, decay of light can be observed as a result of the presence of strong scattering, when the mean free path of the light is comparable to the wavelength of light, which is the regime of Anderson localization [10]. If the dimensionality of the system is larger than one, the reflected light will be scattered. However, if the decay of the light is due to a photonic bandgap (PBG), light should be reflected ballistically (for frequencies below the diffraction cut-off).

Another example of a stopband in the absence of a PBG is when incident light cannot excite a Bloch state of particular symmetry, for example a plane wave incident on a hexagonal photonic crystal, which has 'chess-board'-like states for the Γ–K direction in the TM polarization. In this case the light will be reflected back from the sample, despite there being no PBG. However, the dependence of transmission coefficient on sample thickness will not be an exponential decay and the spectral shape of the transmission dip in a semi-log plot will not be parabolic-like.

In the experimental studies, there are two main schemes which analyse either ballistic transmission (reflection) when the component of the wavevector parallel to the sample boundary does not change, or total transmission (reflection), which is the sum of all the waves emerging from the rear (reflected from the front) side of the sample. Total transmission (reflection) measurements include both the ballistic and scattered waves. Recent studies of photonic crystals have shown the distinction between reflection and transmission spectra in the case of photonic crystals with *complete* (omnidirectional) and *incomplete* (sometimes referred to as 'pseudo-gap') bandgaps. For a stopband that is below the diffraction cut-off, ballistically transmitted light dominates the spectrum in the case of a *complete* PBG [11], while in the case of an *incomplete* PBG the intensity of the scattered transmitted light is much larger within the stopband [12].

To investigate either of the stopbands observed in photonic quasicrystals, corresponding to a complete photonic bandgap, we propose to analyse the directionality of the reflected and transmitted light.

The quasicrystalline structure considered is obtained by tiling a plane with two kinds of rhombus: a thin tile (with vertex angles 36° and 144°) and a fat tile (72° and 108°), as shown in figure 1. Air cylinders, placed at the vertices of the tiles, form the physical structure. The radius of the air cylinder is $0.17d$, where d is the side length of a tile.

In order to investigate the influence of the refractive index contrast on the properties of quasicrystals, we study two structures, one with material refractive index $n = 1.45$ (corresponding to silica), subsequently referred to as the 'low index structure', and one with refractive index of 3.0 (similar to that of III–V semiconductors), referred to as the high index structure. Figure 2 shows the Fourier representation of the dielectric contrast in the quasicrystal under study. The Fourier representation and the diffraction pattern corresponding to the experimental structure [8] possesses tenfold rotational symmetry. Three intense series of diffraction peaks dominate the diffraction pattern. To index the diffraction pattern we choose the set of reciprocal lattice vectors (RVs) corresponding to the internal set of these series of diffraction spots as a basic set of RVs, denoted by $\pm\mathbf{F}_i$ ($i = 1, 5$), where

$$
\begin{aligned}
\mathbf{F}_1 &= (1, 0) = (10000)\\
\mathbf{F}_2 &= (\cos(\pi/5), \sin(\pi/5)) = (01000)\\
\mathbf{F}_3 &= (\cos(2\pi/5), \sin(2\pi/5)) = (00100)\\
\mathbf{F}_4 &= (\cos(3\pi/5), \sin(3\pi/5)) = (00010)\\
\mathbf{F}_5 &= (\cos(4\pi/5), \sin(4\pi/5)) = (00001).
\end{aligned}
$$

The magnitude of each $\mathbf{F}_i$ is related to the tile side length d by $|\mathbf{F}_i| = 2\pi/(d\cos(\pi/10)) \approx 1.051 \times 2\pi/d$ which corresponds to the inverse of the long diagonal of the thin tile. Using this notation the three major series of diffraction peaks dominating the diffraction pattern can be defined as (10000), (10010) and (01100).

If a light wave propagates in media for which the spatially varying dielectric constant has a substantial distinct Fourier coefficient related to some RV $\mathbf{g}$, the wave can be reflected strongly if the magnitude of its wavevector $|\mathbf{K}|$ (which is approximately $\langle n\rangle\omega/c$ where n is the mean refractive index) is equal to $|\mathbf{g}|/2$. For the structure under study, with $\langle n\rangle \approx 1.4$, the

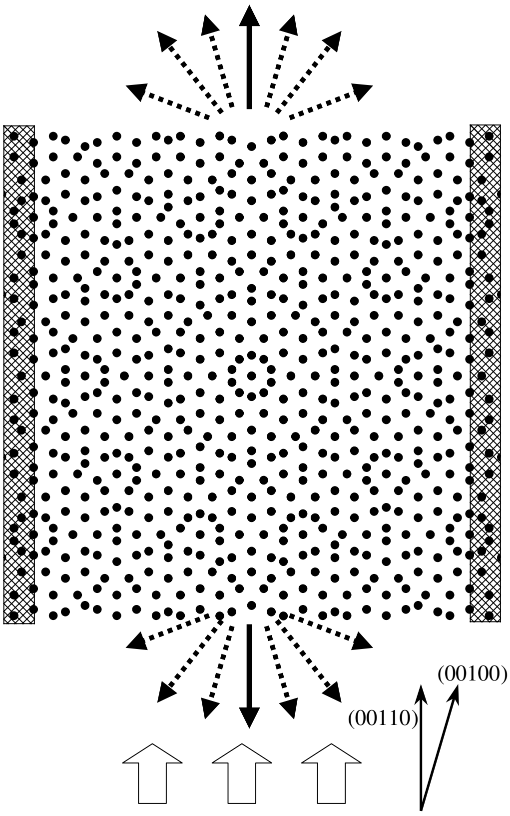

Figure 1. Schematic view of the Penrose structure, obtained using thin and thick tiles. The black circles placed at the vertices of the tiling correspond to air cylinders. The hatched areas indicate the left- and right-hand boundaries, where periodic boundary conditions (required for the transmission calculation) can be set up without breaking the tiling near the boundary.

centre frequencies f of a the possible gaps should satisfy the relation

$$fd/c \approx \frac{1.05}{2\langle\sqrt{n}\rangle}\frac{\mathbf{g}}{\mathbf{F}} \tag{1}$$

which in the case of the low index structure gives the value $fd/c \approx 0.37$ for scattering by the (10000) reciprocal vector and the value $fd/c \approx 0.44$ for scattering by the (10100) vector. For the high index structure, the corresponding values are 0.18 and 0.23.

Figure 3 shows the dispersion curves for the photonic quasicrystals obtained by the 'sloped zone approach' [8, 13], which is an analogue of the extended zone scheme for crystals but which utilizes a set of RVs corresponding to the strongest peaks in the diffraction pattern. The dispersion curves shown are obtained using just 30 reciprocal lattice vectors corresponding to the strongest diffraction peaks, namely RVs such as (10000), (10100) and (11000), together with (0000) describing the average refractive index of the structure. This method is not strictly rigorous but allows us to predict the approximate position of the PBGs in quasicrystals to a good degree of accuracy [8, 13]. It predicts that PBGs should occur near the frequencies $0.36 < fd/c < 0.41$ and $0.44 < fd/c < 0.48$ in the low index structure. These two frequency intervals correspond to Bragg scattering by the (10000) and (10100) reciprocal lattice vectors respectively. Note that the positions of the gap for E-polarized light (when the

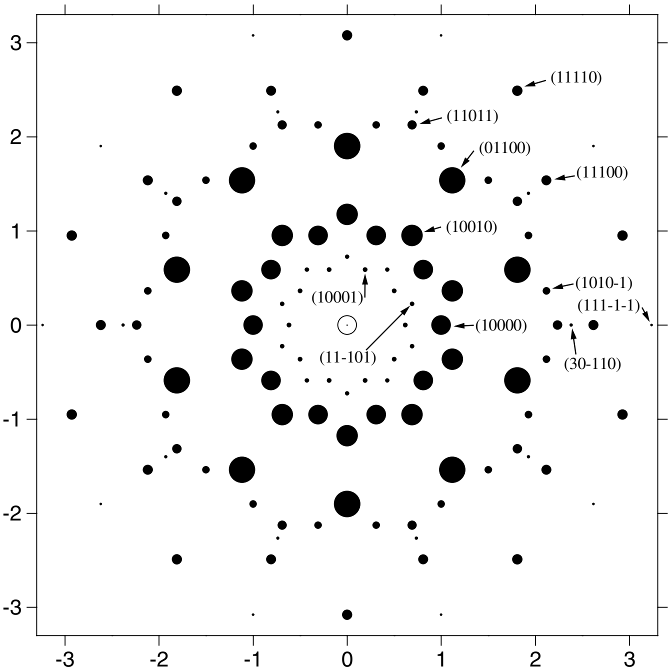

Figure 2. Fourier transform of the dielectric constant distribution in the Penrose tiled quasicrystal. The radius of the spot is proportional to the magnitude of the related Fourier coefficient.

electric field is parallel to the cylinders) and for H-polarized light (when the magnetic field is parallel to the cylinders) are very close. For the high index structure the corresponding frequency intervals with predicted PBGs are $0.19 < fd/c < 0.22$ and $0.23 < fd/c \approx 0.27$.

Figures 4 and 5 show the transmission and reflection spectra for the low index structure for the two symmetry directions (10000) and (01100) respectively. In Penrose-tiled photonic quasicrystals, the degree of rotational symmetry is ten and the angle of 18° between directions (10000) and (01100) is the maximum angular deviation between physically inequivalent directions.

Total reflection (transmission), shown by the solid curves, includes ballistic and scattered waves, shown by the dashed and dotted curves, respectively. It can be seen that for both polarizations there are pronounced peaks in the total reflection spectra accompanied by dips in the total transmission spectra. These spectral features are centred at frequencies $fd/c \approx 0.367$ and $fd/c \approx 0.456$, for both directions of propagation and both polarizations, corresponding well to our approximate estimates made by the 'sloped zone approach'. The relative widths of the lower and higher frequency dips are about 1.5% and 4%, respectively.

Outside the stopbands, the total transmission coefficient is close to unity and the intensity of ballistic light is much larger than that of the scattered light despite the large ($20d$) thickness of the structure. This suggests that, despite the structure having no periodicity, propagation of light inside the structure is provided by Bloch-like states, and not through the random diffusion of photons. Also, the scattered transmitted intensity increases with an increase of the frequency. Note that there is an increase of the intensity of scattered light in transmission near the higher frequency stopband, centred at frequency $fd/c \approx 0.46$. For the lower frequency stopband, centred at $fd/c \approx 0.37$, an increase of the scattered intensity in the transmission does occur.

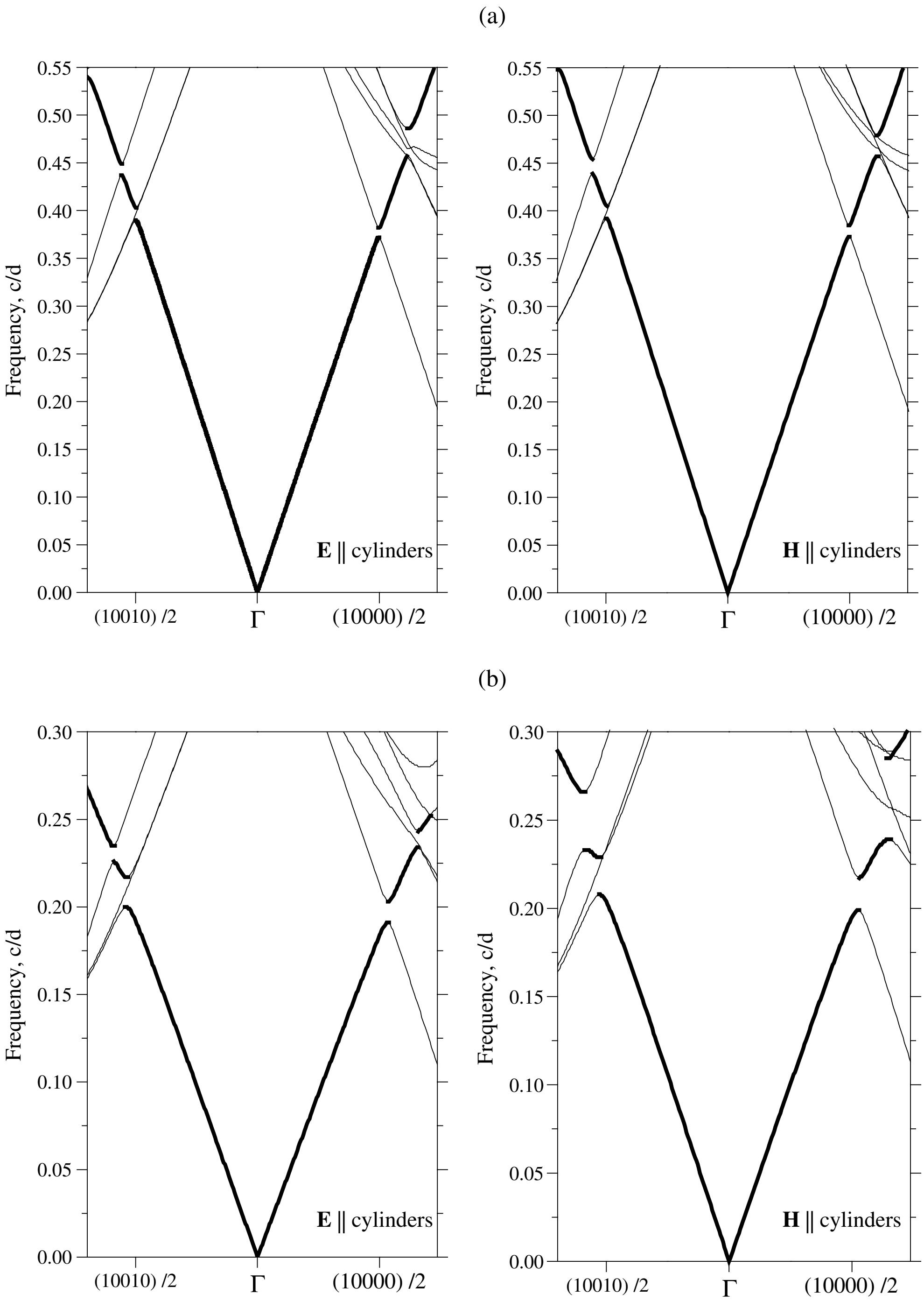

Figure 3. Dispersion relations for photons in the Penrose tiled phonic quasicrystal obtained using the major sets of reciprocal vectors such as (10000), (10100), and (11000). Thick lines highlight the branches oriented along the free photon line. (a) Low index structure; (b) high index structure.

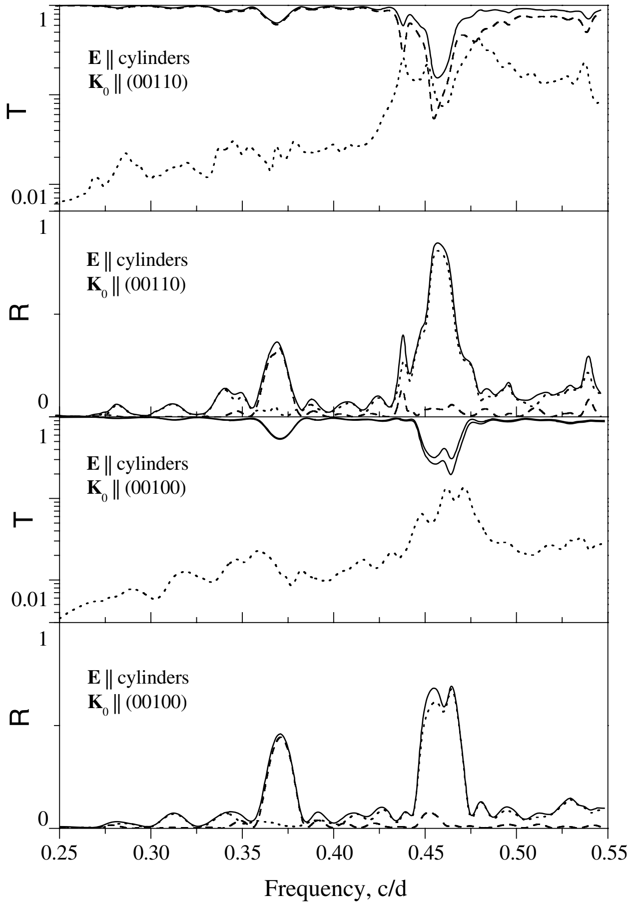

Figure 4. Transmission and reflection spectra of the E-polarized waves for the low index structure along (00110), perpendicular to the boundary of the structure, and along (10000), 18° off the normal to the boundary of the structure, as shown in figure 1. Dashed curves correspond to ballistic light while dotted curves correspond to scattered light. Total transmission (reflection), which is the sum of ballistic and scattered light, is shown by the solid curves. The frequency intervals corresponding to the low and high frequency gaps are $0.364 < fd/c < 0.37$ and $0.448 < fd/c < 0.467$ respectively.

The most interesting feature of the reflection spectra is the fact that the low frequency peak at $fd/c \approx 0.37$ is purely ballistic, while for the higher frequency peak at $fd/c \approx 0.46$ the light is scattered.

For the low index structure, attenuation of light through the full thickness of the structure is not large enough to consider photonic bandgap effects. However, in the high index structure the reflection coefficient reaches the value of 0.9 as can be seen in figures 6 and 7 and one can say that a PBG is established. Note that the positions of the gaps observed do not depend on the sample thickness while the transmission of light decreases with an increase of the sample thickness [8].

The relative width of the stopbands for a particular direction of propagation and polarization increases by a factor of two compared to the low index structure, but some shifts of the stopbands do occur. Nevertheless, the stopbands for both directions of propagation and polarizations do overlap. For the low frequency gap appearing due to diffraction by RV

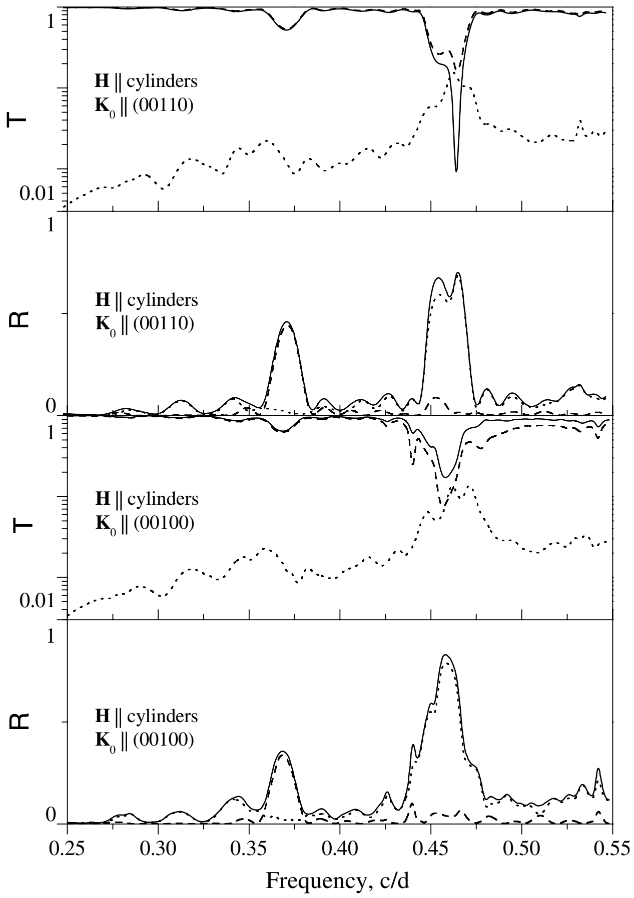

Figure 5. Transmission and reflection spectra of the H-polarized waves for the low index structure along (00110), perpendicular to the boundary of the structure, and along (10000), 18° off the normal to boundary of the structure, as shown in figure 1. Dashed curves correspond to ballistic light while dotted curves correspond to scattered light. Total transmission (reflection) which is the sum of ballistic and scattered light is shown by the solid curves. The frequency intervals corresponding to the low and high frequency gaps are $0.364 < fd/c < 0.37$ and $0.448 < fd/c < 0.467$ respectively.

(10000) the total PBG is centred at $fd/c \approx 0.182$ and has a relative width 2%, while for the high frequency gap appearing due to diffraction by RV (10010) the total PBG is centred at $fd/c \approx 0.227$ and has a relative width of 6%.

For practical applications the relative width of the PBG should exceed 10%, which is not the case for the systems under consideration. Nevertheless, the relative width of the PBG and the extinction coefficient can be increased by the use of a pair of materials with higher refractive index contrast or by increasing the filling factor. Also, in photonic quasicrystals with an octagonal arrangement of cylinders [6], the relative width of the PBG is larger than that for Penrose-tiled quasicrystals, due to the larger magnitude of the Fourier coefficient of the dominant RV.

In a similar way to the case of the low index structure, light is reflected ballistically for the low frequency stopband, while for the higher frequency stopband reflected light is scattered. This property of the reflection spectra can be easily explained. Neglecting scattering by the

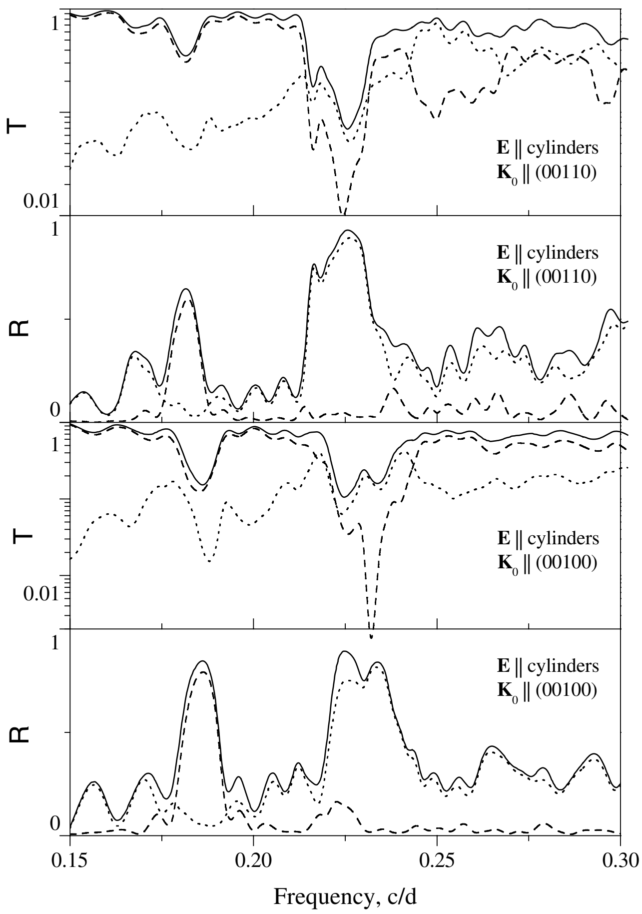

Figure 6. Transmission and reflection spectra of the E-polarized waves for the high index structure along (00110), perpendicular to the boundary of the structure, and along (10000), 18° off the normal to boundary of the structure, as shown in figure 1. Dashed curves correspond to ballistic light while dotted curves correspond to scattered light. Total transmission (reflection) which is the sum of ballistic and scattered light is shown by the solid curves. The frequency intervals corresponding to the low and high frequency gaps are $0.18 < fd/c < 0.185$ and $0.22 < fd/c < 0.233$ respectively.

(10001) RV, which has a very small amplitude of Fourier coefficient (see figure 2), the lower frequency stopband, which originates from diffraction by the (10000) RV, can be considered to be below the diffraction cut-off [14][1]. In contrast, the high frequency stopband originates from diffraction by the RV (10010), and light at the frequency corresponding to the high frequency stopband experiences diffraction by RV (10000) and thus is scattered.

An interesting feature of the transmission spectra is the fact that in the frequency region near the low frequency stopband scattered transmission is much smaller than its ballistic counterpart, and within the low frequency stopband this is further reduced. Such behaviour constitutes clear evidence that the system possesses a complete omnidirectional PBG [11].

[1] The Fourier transform of the dielectric contrast of the quasicrystals is of a fractal nature: sharp diffraction peaks densely fill all reciprocal space (see Janssen (1998) in [14]). This is one reason for the appearance of scattered light below the diffraction cut-off. The other reason is that the periodic mesh used in the calculation scheme (see Bell *et al* (1995) in [14]) does not match the quasiperiodic arrangement of the cylinders in the system under study.

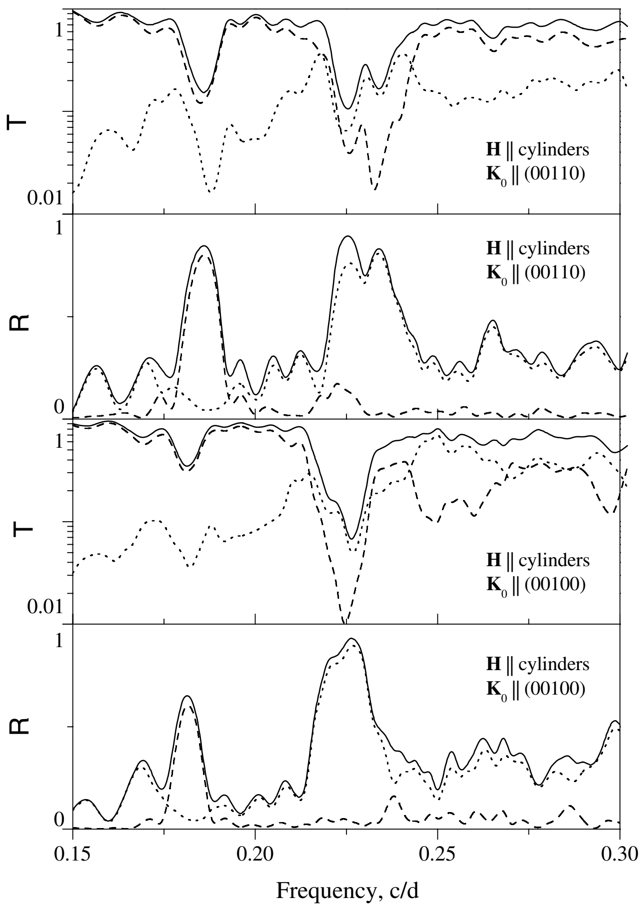

Figure 7. Transmission and reflection spectra of the H-polarized waves for the high index structure along (00110), perpendicular to the boundary of the structure, and along (10000), 18° off the normal to boundary of the structure, as shown in figure 1. Dashed curves correspond to ballistic light while dotted curves correspond to scattered light. Total transmission (reflection) which is the sum of ballistic and scattered light is shown by the solid curves. The frequency intervals corresponding to the low and high frequency gaps are $0.18 < fd/c < 0.185$ and $0.221 < fd/c < 0.234$ respectively.

2. Conclusions

By analysing the nature of the reflected and transmitted light and directionality of their spectra we have demonstrated that the Penrose-type photonic quasicrystals studied possess complete omnidirectional photonic bandgaps, for both E and H polarizations.

Acknowledgment

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